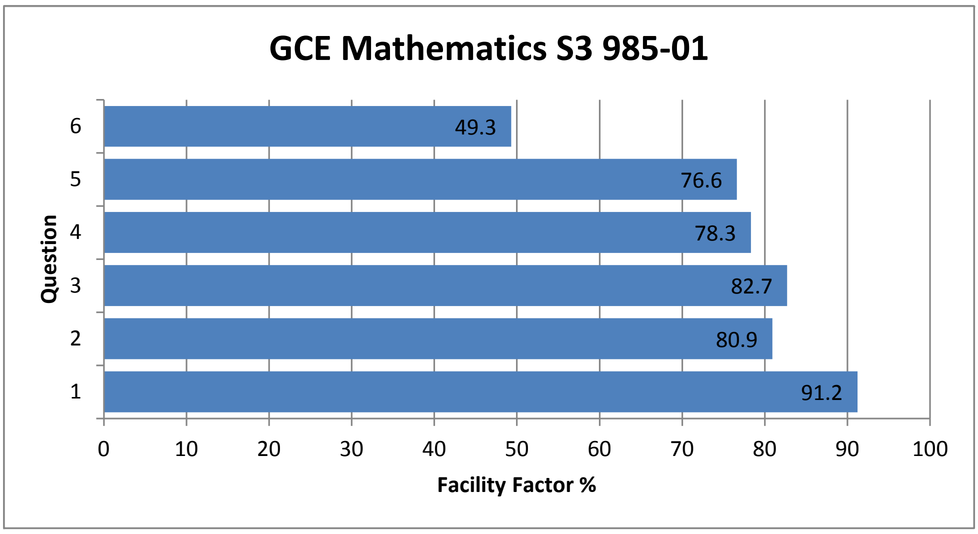
# WJEC 2014 Online Exam Review

## GCE Mathematics S3 985-01

**All Candidates' performance across questions**

| Question Title | N | Mean | S D | Max Mark | F F | Attempt % | |
|---|---|---|---|---|---|---|---|
| 1 | 174 | 5.5 | 1.2 | 6 | 91.2 | 100 | |
| 2 | 174 | 10.5 | 2.6 | 13 | 80.9 | 100 | 2 |
| 3 | 173 | 10.7 | 3.1 | 13 | 82.7 | 99.4 | |
| 4 | 174 | 8.6 | 2.6 | 11 | 78.3 | 100 | 4 |
| 5 | 174 | 13.8 | 4.1 | 18 | 76.6 | 100 | 5b ii and iii |
| 6 | 171 | 6.9 | 4.7 | 14 | 49.3 | 98.3 | 6 |



**2.** The mean weight of a certain breed of bird is believed to be 4·5 kg. In order to test this belief, a random sample of 10 birds of the breed was obtained and weighed, with the following results in kg.

4·38 4·18 4·46 4·59 4·16 4·57 4·16 4·26 4·49 4·35

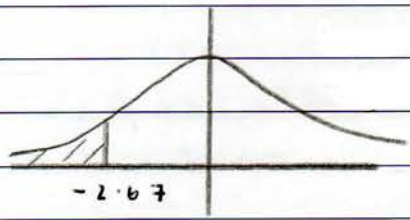
You may assume that the weights of this breed of bird are normally distributed.

*(a)* State suitable hypotheses for testing the above belief using a two-sided test. [1]

*(b)* Calculate unbiased estimates of the mean and the variance of the weights of this breed of bird. [5]

*(c)* Carry out an appropriate test using a 1% significance level and state your conclusion in context, justifying your answer. [7]

| Question number<br>*Rhif y Cwestiwn* | | Leave Blank<br>*Gadewch yn wag* |
|---|---|---|
| **2.** | $p$ value $= 2 \times P[\bar{x} \le 4.36]$<br>$= 2 \times P\left[Z \le \dfrac{4.36 - 4.5}{0.0523662317}\right]$<br>$= 2 \times P[Z \le -2.67]$<br><br>$= 2 \times P[Z \geq 2.67]$<br>$= 2 \times 0.00379$<br>$= \underline{0.00758}$<br>$p$ value $< 0.01$ so very strong evidence to reject $H_0$ ie we would accept the mean weight of a certain breed of bird $\neq 4.5$ kg. | |

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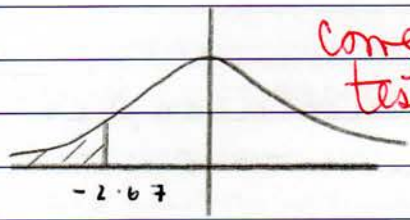
**2.** p value $= 2 \times P[\bar{X} \le 4.36]$ M1

$= 2 \times P\left[Z \le \frac{4.36 - 4.5}{0.0523662317}\right]$ A1

$= 2 \times P[Z \le -2.67]$ A1

correct test-stat




$= 2 \times P[Z \ge 2.67]$ B0

$= 2 \times 0.00379$ t not Z B0

$= \underline{0.00758}$ B0

B0

p value $< 0.01$ so very strong evidence to reject $H_0$ ie we would accept the mean weight of a certain breed of bird $\ne 4.5$ kg. X

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**4.** A market gardener grows a large number of tomato plants, all of which are Variety A or Variety B. He wishes to investigate whether or not there is a difference in the mean weights of tomatoes obtained from these two varieties over the whole season. He therefore selects random samples of 80 plants of Variety A and 70 plants of Variety B and he records the total yield, in kg, from each plant. At the end of the season, he produces the following summary statistics.

| | Variety A | Variety B |
|---|---|---|
| Sample size | 80 | 70 |
| Sample mean | 3·52 | 3·65 |
| Unbiased variance estimate | 0·115 | 0·096 |

*(a)* State suitable hypotheses for the investigation. [1]

*(b)* Calculate the approximate *p*-value of the above results and state your conclusion in context. [8]

*(c)* Give two reasons why the *p*-value is approximate and not exact. [2]

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4.

$$\therefore \text{p-value} = 2P(\bar{Y} - \bar{X} > 0.13)$$

$$= 2P\left(Z > \frac{0.13 - 0}{\sqrt{\frac{0.115}{80} + \frac{0.096}{70}}}\right)$$

$$= 2P(Z > 2.45286)$$

$$\approx 2P(Z > 2.45)$$

$$= 2[1 - P(Z < 2.45]$$

$$= 2[1 - 0.99286]$$

$$= 0.01428 \quad < 0.05$$

$\therefore$ there is strong evidence to reject $H_0$. therefore there is a difference in the wieghts of variety A and B of tomato plants.

c) The p-value is approximate and not exact because
- variance has been estimated
- and CLT assumes Normal population

CLT = Central Limit Theorem.

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QUES 4

4. $\therefore$ p-value $= 2P(\bar{Y}-\bar{X} > 0.13)$

$$= 2P\left(Z > \frac{0.13-0}{\sqrt{\frac{0.115}{80}+\frac{0.096}{70}}}\right)$$ M1 A1

$= 2P(Z > 2.45286)$ M1

$\approx 2P(Z > 2.45)$ A1

$= 2[1 - P(Z < 2.45]$

$= 2[1 - 0.99286]$ A1

$= 0.01428 < 0.05$ A1

$\therefore$ there is strong evidence to reject $H_0$.
therefore there is a difference in the mean weights of variety A and B of tomato plants. A1 B0

c) The p-value is approximate and not exact because • variance has been estimated B1
• and CLT assumes Normal population B0
No.

CLT = Central Limit Theorem.

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**5.** The variables $x$ and $y$ are related by an equation of the form $y = \alpha + \beta x$. In order to estimate the unknown constants $\alpha$ and $\beta$, the following measurements were made.

| $x$ | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|
| $y$ | 19·8 | 33·9 | 49·9 | 64·1 | 77·9 | 95·0 |

*(a)* Calculate least squares estimates for $\alpha$ and $\beta$. [8]

*(b)* The values of $x$ are exact but the values of $y$ are subject to independent normally distributed measurement errors with mean zero and standard deviation 0·5.

(i) Calculate an unbiased estimate of the value of $y$ when $x = 5$.

(ii) Determine a 95% confidence interval for the value of $y$ when $x = 5$.

(iii) It was thought beforehand that the value of $\beta$ was 7·6. Determine whether or not, at the 5% significance level, the values in the table above are consistent with this value of $\beta$. [10]

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**5(b).**

ii) $95\%$ CI for $y = \left(41.85 \pm 1.96\sqrt{\frac{0.5^2}{6}}\right)$

$= (41.45\ ,\ 42.25)$

iii) when $x=2$ $y=19.8$

$\beta = 7.625$

$x=4$ average $\beta = 7.47$

$\beta = 7.33$

when $x=6$

$\beta = 7.56$

when $x=8$

$\beta = 7.44$

when $x=10$

$\beta = 7.335$

when $x=12$

$\beta = 7.54$

$P(\beta < 7.47) = P\left(Z < \frac{7.47 - 7.6}{0.5}\right)$

$= P(Z > 0.26)$

$= 0.3974$

at 5% sig level values of $\beta$ are not consistent

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**5(b).**

ii) 95% CI for $y = \left(41.85 \pm 1.96\sqrt{\frac{0.5^2}{6}}\right)$ M0

$= (41.45\ ,\ 42.25)$ A0

m0

iii) when $x = 2$ $y = 19.8$

$\beta = 7.625$

$x = 4$ average $\beta = 7.47$

$\beta = 7.37$

when $x = 6$

$\beta = 7.56$

when $x = 8$

$\beta = 7.44$

when $x = 10$

$\beta = 7.335$

when $x = 12$

$\beta = 7.54$

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$P(\cancel{x}\,\beta < 7.47) = P\left(\cancel{z} < \frac{7.47 - 7.6}{0.5}\right)$ M0

$= P(\cancel{z} > 0.26)$ ✗

$= 0.3974$

at 5% sig level values of $\beta$ are not consistent

✗ B0

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**6.** The continuous random variable $X$ is uniformly distributed on the interval [0, $\theta$], where $\theta$ is unknown. In order to estimate $\theta$, a random sample of $n$ observations on $X$ is obtained and $\overline{X}$ denotes the mean of this sample. An unbiased estimator for $\theta$ is given by $Y = k\overline{X}$.

*(a)* (i) Find the value of $k$.

(ii) Find the standard error of $Y$. [8]

*(b)* (i) Show that $Y^2$ is not an unbiased estimator for $\theta^2$.

(ii) Find an unbiased estimator for $\theta^2$. [6]

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6.a)i) $E(\bar{X}) = E(X) = \frac{\theta}{2}$

$E(Y) = \frac{k\theta}{2} = \theta$ if $k=2$

ii) $Var(Y) = 4\,Var(\bar{X}) = 4 \times \frac{\theta^2}{12n} = \frac{\theta^2}{3n}$

$SE$ of $Y = \sqrt{\frac{\theta^2}{3n}} = \frac{\theta}{\sqrt{3n}}$

b)i) $Y^2 = k^2\bar{X}^2$ $\qquad E(\bar{X}^2) = E(X^2)$

$E(Y^2) = 4E(\bar{X}^2)$

$E(X^2) = Var(X) + E(X)^2$

$= \frac{\theta^2}{12} + \frac{\theta^2}{4} = \frac{\theta^2}{3}$ $\qquad E(Y^2) = \frac{4\theta^2}{3} \neq \theta^2$

ii) UE of $\theta^2 = 3\bar{X}^2$

$Var(\bar{X}) = \frac{\sigma^2}{n} = \int$

$f(x) = \frac{1}{\theta}$

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6.(a)(i) $E(\bar{X}) = E(X) = \frac{\theta}{2}$

$E(Y) = \frac{k\theta}{2} = \theta$ if $k=2$ ✓ 3

(ii) $Var(Y) = 4\,Var(\bar{X}) = 4 \times \frac{\theta^2}{12n} = \frac{\theta^2}{3n}$

SE of $Y = \sqrt{\frac{\theta^2}{3n}} = \frac{\theta}{\sqrt{3n}}$ ✓ 5

(b)(i) $Y^2 = k^2\bar{X}^2$  $E(\bar{X}^2) = E(X^2)$

$\cancel{Var(\bar{X}) = \frac{Var(X)}{n}}$  $E(Y^2) = 4E(\bar{X}^2)$ ✓ MO

$\cancel{E(\bar{X}^2) = Var(\bar{X})} = E(X^2) = Var(X) + E(X)^2$  $\bar{x}$ reqd.

$= \frac{\theta^2}{12} + \frac{\theta^2}{4} = \frac{\theta^2}{3}$  $E(Y^2) = \frac{4\theta^2}{3} \neq \theta^2$

(ii) UE of $\theta^2 = 3\bar{x}^2$

$\cancel{Var(\bar{X}) = \frac{\sigma^2}{n}}$

$\cancel{f(x) = \frac{1}{\theta}}$

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